8-th Hungary–Israel Binational Mathematical Competition 1997

April 13–20, 1997

1. Is there an integer N such that
$$\left(\sqrt{1997}-\sqrt{1996}\right)^{1998}=\sqrt{N}-\sqrt{N-1}?$$

2. Find all real numbers α with the following property: for every positive integer n there is a positive integer m such that
$$\left|\alpha-\frac{m}{n}\right|<\frac{1}{3n}.$$

3. Let O be the circumcenter of an acute-angled triangle ABC. The lines AO,BO,CO intersect the opposite sides of the triangle at A_1,B_1,C_1, respectively. Suppose that the circumradius of $\triangle ABC$ is $2p$, where p is a prime number, and that OA_1,OB_1,OC_1 have integer lengths. Find the sides of the triangle.

4. Determine the number of words of the length 1997 formed by the letters A,B,C, where each letter appears an odd number of times.

5. Squares ACC_1A”, ABB_1A', $BCDE$ are contructed in the exterior of a triangle ABC. If P is the center of the square $BCDE$, prove that the lines $A'C$,A”B and PA are concurrent.

6. Can a closed disk be decomposed into a union of two disjoint congruent parts?

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