7-th Hungary–Israel Binational Mathematical Competition 1996

Technion IIT, Israel, March 27, 1996

1. Find all sequences of integers $x_1, x_2, \dots, x_{1997}$ satisfying

$$\sum_{k=1}^{1997} 2^{k-1} x_k^{1997} = 1996 \prod_{k=1}^{1997} x_k.$$

2. Suppose that $n > 2$ is an integer for which n^2 can be written as the difference of the cubes of two consecutive positive integers. Prove that n is the sum of two squares, and show that such an n does exist.

3. Every vertex of a given convex polyhedron is incident with at least four edges. Prove that at least 8 faces of the polyhedron are triangles.

4. Let $a_1, a_2, \dots, a_n$ be arbitrary real numbers and $b_1, b_2, \dots, b_n$ be real numbers with $1 \geq b_1 \geq \dots \geq b_n \geq 0$. Prove that there is a positive integer $k \leq n$ such that

$$|a_1 b_1 + a_2 b_2 + \dots + a_n b_n| \leq |a_1 + a_2 + \dots + a_k|.$$

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