5-th Hungary–Israel Binational Mathematical Competition 1994

1. Let $a_1, a_2, \dots, a_n$ be positive numbers. If $a_{k+1}, \dots, a_n$ $(k < n)$ are fixed, how should one choose $a_1, \dots, a_k$ in order to minimize $\displaystyle\sum_{i \neq j} \frac{a_i}{a_j}$?

2. Three congruent circles pass through a point P are intersect each other again at points A, B, C. The three circles are contained in a triangle $A'B'C'$ whose each side is tangent to two of the circles. Prove that the area of $\triangle A'B'C'$ is at least 9 times the area of $\triangle ABC$.

3. Let m and n be two different natural numbers. Show that there exists a real number x such that $\dfrac{1}{3} \leq \{mx\} \leq \dfrac{2}{3}$ and $\dfrac{1}{3} \leq \{nx\} \leq \dfrac{2}{3}$.

4. An n–m *society* is a group of n girls and m boys. Prove that there exist numbers m_0, n_0 such that every n_0–m_0 society contains a subgroup of five boys and five girls in which every boy knows every girl or no boy knows any girl.

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