1-st Hungary–Israel Binational Mathematical Competition 1990

1. Prove that there are no positive integers x and y such that x^2+y+2 and y^2+4x are perfect squares.

2. In a triangle ABC with $\angle ACB = 90^\circ$, D is the midpoint of BC, E,F the points on AC such that $AE = EF = FC$, CG the altitude and H the circumcenter of triangle AEG. Show that the triangles ABC and HDF are similar.

3. Prove the equality

$$\frac{1989}{2}-\frac{1988}{3}+\frac{1987}{4}-\cdots+\frac{1}{1990}=\frac{1}{996}+\frac{3}{997}+\frac{5}{998}+\cdots+\frac{1989}{1990}.$$

4. At each lattice point of a gridline paper we draw an arrow parallel to one of the sides of the paper (no arrows at the boundary can point outwards). Show that there exist two neighboring points (horizontally, vertically or diagonally) at which the arrows point to opposite directions.

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