28-th German Federal Mathematical Competition 1997/98

Second Round

1. Find all integer solutions (x,y,z) of the equation $xy+yz+zx-xyz=2$.
2. Prove that there exist 16 subsets of set $M=\{1,2,\dots,10000\}$ with the following property: For every $z\in M$ there are eight of these subsets whose intersection is $\{z\}$.
3. A triangle ABC satisfies $BC=AC+\frac{1}{2}AB$. Point P on side AB is taken so that $AP=3PB$. Prove that $\angle PAC=2\angle CPA$.
4. Let $3(2^n-1)$ points be selected in the interior of a polyhedron $\mathscr{P}$ with volume 2^n, where n is a positive integer. Prove that there exists a convex polyhedron $\mathscr{U}$ with volume 1, contained entirely inside $\mathscr{P}$, which contains none of the selected points.

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