25-th German Federal Mathematical Competition 1994/95

Second Round

1. Starting at $(1,1)$, a stone is moved in the coordinate plane according to the following rules:

   (i) From any point (a,b), the stone can move to $(2a,b)$ or $(a,2b)$.

   (ii) From any point (a,b), the stone can move to $(a-b,b)$ if $a>b$, or to $(a,b-a)$ if $a<b$.

   For which positive integers x,y can the stone be moved to (x,y)?

2. Let S be a union of finitely many disjoint subintervals of $[0,1]$ such that no two points in S have distance $1/10$. Show that the total length of the intervals comprising S is at most $1/2$.

3. Each diagonal of a convex pentagon is parallel to one side of the pentagon. Prove that the ratio of the length of a diagonal to that of its corresponding side is the same for all five diagonals, and compute this ratio.

4. Prove that every integer $k>1$ has a multiple less than k^4 whose decimal expension has at most four distinct digits.

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