7-th German Federal Mathematical Competition 1976/77

Second Round

1. Do there exist two infinite sets A and B of nonnegative integers with the property that every nonnegative integer can be uniquely written as a sum of an element of A and an element of B?

2. On a plane are given three non-collinear points A, B, C. We are given a disk of diameter different from that of the circle passing through A, B, C, large enough to cover all the three points. Construct the fourth vertex of the parallelogram $ABCD$ using only this disk. (The disk is to be used as a circular ruler, for constructing a circle passing through two given points.)

3. Show that there are infinitely many positive integers that cannot be represented as $a = a_1^6 + a_2^6 + \cdots + a_7^6$, where a_i are positive integers. State and prove a generalization.

4. A real function f is defined on the set D of rational numbers different from 0 and 1 and satisfies
$$f(x) + f\left(1 - \frac{1}{x}\right) = x \quad \text{for all } x \in D.$$

   Determine f.