

2-nd German Federal Mathematical Competition 1971/72

Second Round

1. A knight is placed on a field of an infinite chessboard. Find the number of fields the knight can reach in n moves.
2. Prove that among any 79 consecutive natural numbers there is one whose sum of digits is divisible by 13. Also show that this statement is not true for 78 numbers.
3. The arithmetic mean of two distinct natural numbers x and y is a two-digit number. If the digits of this number are exchanged, the obtained number is equal to the geometric mean of x and y .
 - (a) Determine x and y .
 - (b) Show that the solution in base $g = 10$ is unique, but that there are no solutions in base $g = 12$.
 - (c) Give further examples of bases g for which there exist such numbers x and y , and those for which there are no such numbers.
4. There are p participants in a chess tournament, and any two of them play at most one match against each other. After n matches are played, in each group of three players there exist two who have not yet played against each other. Prove that $n \leq p^2/4$.