29-th German Federal Mathematical Competition 1998/99

First Round

1. 1600 coconuts are distributed over 100 monkeys, allowing some monkeys to be left empty-handed. Prove that, independent of the distribution, there always exist four monkeys with the same number of coconuts.

2. The sequences (a_n) and (b_n) are defined by $a_1 = b_1 = 1$ and

$$a_{n+1} = a_n + b_n, \quad b_{n+1} = a_n b_n \quad \text{for } n = 1, 2, \ldots$$

   Show that every two distinct terms of the sequence (a_n) are coprime.

3. In the plane are given a segment AC and a point B on the segment. Let us draw the positively oriented isosceles triangles ABS_1, BCS_2, and CAS_3 with the angles at S_1, S_2, S_3 equal to $120°$. Prove that the triangle $S_1 S_2 S_3$ is equilateral.

4. It is known that there are polyhedrons whose faces are more numbered than the vertices. Find the smallest number of triangular faces that such a polyhedron can have.