26-th German Federal Mathematical Competition 1995/96

First Round

1. Can a square of side length 5 be covered by three squares of side length 4?

2. The cells of an $n \times n$ board are labelled with the numbers 1 through n^2 in the usual way. Let n of these cells be selected, no two of which are in the same row or column. Find all possible values of the sum of their labels.

3. Four lines are given in a plane so that any three of them determine a triangle. One of these lines is parallel to a median in the triangle determined by the other three lines. Prove that each of the other three lines also has this property.

4. Find all positive integers n for which $n \cdot 2^{n-1} + 1$ is a perfect square.