11-th German Federal Mathematical Competition 1980/81

First Round

1. Let a and n be positive integers and $s = a + a^2 + \cdots + a^n$. Prove that the last digit of s is 1 if and only if the last digits of a and n are both equal to 1.

2. Prove that if the sides a, b, c of a non-equilateral triangle satisfy $a + b = 2c$, then the line passing through the incenter and the circumcenter is parallel to one of the sides of the triangle.

3. A square of side 2^n is divided into unit squares. One of the unit squares is cut off. Prove that the rest of the square can be tiled with tiles of the form  .

4. Prove that if p is a prime number, then $2^p + 3^p$ is not of the form n^k, where n and $k > 1$ are positive integers.

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