36-th German Federal Mathematical Competition 2005/06

First Round

1. Find two consecutive natural numbers each of which has the sum of digits divisible by 2006.

2. Prove that the equation $x^3+y^3=4(x^2y+xy^2+1)$ has no integer solutions.

3. The sides a,b,c of a triangle satisfy $a^2+b^2>5c^2$. Prove that c is the shortest side of this triangle.

4. There is a square sheet of paper on the table. It is cut into several parts by repeating the following operation: One of the parts is taken and cut into two parts by a straight cut, and the two obtained parts are put back on the table. Find the smallest number of cuts required to obtain at least one hundred 20-gonal parts.

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