35-th German Federal Mathematical Competition 2004/05

First Round

1. At the central square of a 2005×2005 chessboard there is a die. The die is moved on the board by a sequence of moves. Each move consists of three steps: (1) The die is first rolled without moving so that an arbitrary face is on the top, then (2) it is moved right or left for the number of squares shown on its top face, and finally, (3) it is moved up or down for the number of squares shown on its bottom face. Which squares of the board can the die reach?

2. An integer a has the property that $3a$ can be written in the form $x^2 + 2y^2$ for some integers x, y. Show that a can be written in this form also.

3. The sides a, b, c of a triangle correspond to the angles α, β, γ respectively. Prove that if $3\alpha + 2\beta = 180°$, then $a^2 + bc = c^2$.

4. For which positive integers n can the numbers $1, 2, \ldots, n$ be arranged in a sequence in such a way that for no two of the numbers in the sequence does their arithmetic mean lie between them?