

# 34-th German Federal Mathematical Competition 2003/04

## First Round

1. At the beginning of a game the numbers  $1, 2, \dots, 2004$  are written on a blackboard. A move consists of selecting an arbitrary set of numbers on the board and replacing these numbers by the remainder of their sum upon division by 11. At the end of the game two single numbers were left over, one of them being 1000. Find, with proof, the other number.
2. The side lengths  $a, b, c$  of a triangle are integers. Moreover, the length of one altitude equals the sum of the lengths of the other two altitudes. Prove that  $a^2 + b^2 + c^2$  is a perfect square.
3. Show that two congruent regular hexagons can be cut into six parts in total which can be reassembled into an equilateral triangle (without gaps and overlapping).
4. A cube is dissected into a finite number of rectangular parallelepipeds such that the volume of the circumsphere of the cube equals the sum of the volumes of the circumspheres of all rectangular parallelepipeds. Prove that all the parallelepipeds are cubes.