32-nd German Federal Mathematical Competition 2001/02

First Round

1. On planet Ypsilon a year has 365 days – like on Earth. Each month has 28, 30 or 31 days. Prove that a year on Ypsilon also has 12 months.

2. Each ticket of a lottery has a nine-digit serial number formed by digits 1, 2 and 3. A ticket can be red, blue, or yellow. Two tickets which are different at each of the nine places always have different colors. Somebody draws a red ticket numbered 122 222 222 and a yellow ticket numbered 222 222 222. The first prize goes to the ticket numbered 123 123 123. Which color does it have?

3. A circle is dissected into eight arcs by the sides of a convex quadrilateral, with four arcs inside and four arcs outside the quadrilateral (see Figure). Suppose that the lengths of the arcs a, b, c, d satisfy $a + c = b + d$. Prove that the quadrilateral is cyclic.
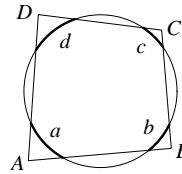

Figure 1:

4. A 12-gon is composed of twelve segments of lengths $1, 2, 3, \ldots, 12$. Prove that the 12-gon always has three consecutive sides whose total length is greater than 20.