Finnish High School Mathematical Contest 1998

Final Round

January 30, 1998

1. Show that it is possible to place points A, B, C, D in the plane so that the area of the quadrangle $ABCD$ is twice the area of the quadrangle $ADBC$.

2. There are 11 members in a competition committee. The competition problems are kept in a room secured by several locks. The keys to the locks are distributed to the committee members in such a way that any six members, but no five members, can open the locks. What is the minimal number of locks needed, and how many keys should each committee member be given?

3. Is it possible to choose a geometric sequence (finite or infinite) from the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$ whose sum of terms is $\frac{1}{5}$?

4. Let be given 110 points inside a unit circle. Show that at least four of these points lie inside a circle of radius $1/8$.

5. Several 5×5 and 7×7 squares are put on a 15×36 grid without overlapping. How many unit squares can be covered?

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