Finnish High School Mathematical Contest 2001

Final Round

February 2, 2001

1. In a right-angle triangle ABC, AB is the hypotenuse and CF an altitude. The circle through F centered at B and the circle of the same radius centered at A have a common point on the side BC. Determine the ratio $FB : BC$.

2. The parabolas $y = ax^2 + bx + c$ and $y = dx^2 + ex + f$ have no common points, where a, b, c, d, e, f are real numbers with $ad < 0$. Prove that there is a line having no common points with either of the two parabolas.

3. Positive integers a, b, c satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq 1$. Prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{41}{42}$.

4. In the weekly State Lottery, a sequence of seven numbers from the set $\{0, 1, \ldots, 9\}$ is picked at random. Compute the probability that the sequence contains exactly five distinct numbers.

5. Determine all positive integers n such that $n^2 + 2$ divides $2001n + 2$.

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