Finnish High School Mathematical Contest 2000

Final Round

January 28, 2000

1. Two circles touch each other externally at A and touch a common tangent at B and C ($B \neq C$). Let D and E be the points such that BD and CE are diameters of the circles. Prove that points D, A, C are collinear.

2. Prove that $\left[(3+\sqrt{5})^n\right]$ is odd for every positive integer n.

3. Find all positive integers n such that $n! > \sqrt{n^n}$.

4. On the plane are given seven points, no three collinear. Any two points are joined by a blue or red segment. Show that there are at least four monochromatic triangles in the obtained figure.

5. Irja and Valtteri both have a counter, and the corners are initially placed in the opposite corners of a square. They alternately move their corners by flipping a coin, with Irja starting. A person who gets the head moves his/her counter to the opposite corner, while a person who gets the tail moves the counter to the adjacent corner, where Irja moves her counter counterclockwise, and Valtteri clockwise. The person whose counter is first moved to the corner occupied by the opponent's counter wins. What is the probability for Irja to win?

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com