Croatian Team Selection Test 2002

Zadar, May 4

1. A language uses n letters. A sequence of letters is a word if and only if there are no two equal letters between two other equal letters. Find the number of words of the maximum length in this language.

2. A quadrilateral $ABCD$ is circumscribed about a circle. Lines AC and DC meet at point E and lines DA and BC meet at F, where B is between A and E and between C and F. Let I_1, I_2 and I_3 be the incenters of triangles AFB, BEC and ABC, respectively. The line I_1I_3 intersects EA at K and ED at L, whereas the line I_2I_3 intersects FC at M and FD at N. Prove that $EK = EL$ if and only if $FM = FN$.

3. Prove that if n is a natural number such that $1 + 2^n + 4^n$ is prime then $n = 3^k$ for some $k \in \mathbb{N}_0$.