Chinese IMO Team Selection Test 2004

Time: 4.5 hours each day.

First Day – Guangzhou, March 31

1. Let P be a point inside a right angle XOY such that $OP = 1$ and $\angle XOP = \pi/6$. A line through P meets the rays OX and OY at M and N. Find the maximum value of $OM + ON - MN$.

2. Let u be a fixed integer. Prove that the equation $u^a - u^b = n!$ has finitely many solutions (a,b,n) in positive integers.

3. Suppose that positive integers $1 < n_1 < n_2 < \cdots < n_k$ $(k \geq 2)$ and $a, b \in \mathbb{N}$ satisfy

$$\prod_{i=1}^{k}\left(1 - \frac{1}{n_i}\right) \leq \frac{a}{b} < \prod_{i=1}^{k-1}\left(1 - \frac{1}{n_i}\right).$$

   Prove that $n_1 n_2 \cdots n_k \leq (4a)^{2^k - 1}$.

Second Day – Guangzhou, April 1

4. Let D, E, F be points on sides BC, CA, AB respectively of a triangle ABC such that $EF \parallel BC$. Let D_1 be an arbitrary point on BC and $E_1 \in CA$, $F_1 \in AB$ be such that $D_1E_1 \parallel DE$ and $D_1F_1 \parallel DF$. Let P be a point on the same side of BC as point A such that $\triangle PBC \sim \triangle DEF$. Prove that lines EF, E_1F_1 and PD_1 are concurrent.

5. Let $p_1, p_2, \dots, p_{25}$ be primes smaller than 2004. Find the largest $T \in \mathbb{N}$ such that every positive integer not exceeding T can be expressed as a sum of distinct divisors of $(p_1 p_2 \dots p_{25})^{2004}$.

6. Let a, b, c be sides of a triangle whose perimeter does not exceed 2π. Prove that $\sin a, \sin b, \sin c$ are also sides of a triangle.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com