21-st Canadian Mathematical Olympiad 1989

April 5, 1989

1. The numbers $1, 2, \ldots, n$ are placed in order so that each number is either strictly bigger or strictly smaller than all the preceding values. In how many ways can this be done?

2. Let ABC be a right-angled triangle of area 1, and let A', B', C' be the reflections of A, B, C in their opposite sides, respectively. Find the area of $\triangle A'B'C'$.

3. The sequence (a_n) is defined as follows: $a_1 = 1989^{1989}$ and, for $n > 1$, a_n is the sum of the digits of a_{n-1}. What is the value of a_5?

4. There are 5 monkeys and 5 ladders and at the top of each ladder there is a banana. A number of ropes connect the ladders, where each rope connects two ladders and no two ropes are attached to the same rung of the same ladder. Each monkey starts at the bottom of a different ladder. The monkeys climb up the ladders but each time they encounter a rope they climb along it to the ladder at the other end of the rope and then continue climbing upwards. Show that, no matter how many ropes there are, each monkey gets a banana.

5. For each permutation $\sigma(X_1, X_2, \ldots, X_n)$ of the numbers $1, 2, 2^2, \ldots, 2^{n-1}$, we define
$$S_i(\sigma) = X_1 + X_2 + \cdots + X_i \quad \text{for each } i = 1, 2, \ldots, n$$
and
$$Q(\sigma) = S_1(\sigma)S_2(\sigma)\cdots S_n(\sigma).$$
Evaluate $\sum \frac{1}{Q(\sigma)}$, where the sum is taken over all possible permutations.

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