19-th Canadian Mathematical Olympiad 1987

1. Find all solutions of $a^2+b^2=n!$ in positive integers a,b,n with $a\leq b$ and $n<14$.

2. The number 1987 can be written as a three digit number $\overline{xyz}$ in some base b such that $x+y+z=1+9+8+7$. Find all possible values of x,y,z,b.

3. Let $ABCD$ be a parallelogram and E be a point on BC between B and C. If the triangles DEC, BED and BAD are isosceles, what are the possible values of $\angle DAB$?

4. On a large, flat field n people are positioned so that for each person the distances to all the other people are different. Each person holds a water pistol and at a given signal fires and hits the person who is closest. When n is odd, show that there is at least one person left dry. Is this always true when n is even?

5. For every positive integer n show that

$$\left[\sqrt{n}+\sqrt{n+1}\right]=\left[\sqrt{4n+1}\right]=\left[\sqrt{4n+2}\right]=\left[\sqrt{4n+3}\right].$$

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