# 18-th Canadian Mathematical Olympiad 1986

May 7, 1986

1. Suppose that there is a point $C$ on the side $AD$ of a triangle $ABD$ such that $\angle ABC = 90^\circ$, $\angle CBD = 30^\circ$ and $AB = CD = 1$. Find $AC$.

2. A Mathlon is a competition in which there are $M$ athletic events. Only teams $A, B, C$ participate. In each event $p_1$ points were awarded for first place, $p_2$ for second and $p_3$ for third, where $p_1 > p_2 > p_3 > 0$ are integers. The final score for $A$ was 22, and for $B$ and $C$ was 9. It is known that $B$ won the 100 meters. What is the value of $M$ and who was the second in the high jump?

3. A chord $ST$ of constant length slides around a semicircle with diameter $AB$. Let $M$ be the midpoint of $ST$ and $P$ be the projection of $S$ to $AB$. Prove that $\angle SPM$ is constant for all positions of $ST$.

4. For positive integers $n, k$, define $F(n,k) = \sum_{r=1}^{n} r^{2k-1}$. Prove that $F(n,1)$ divides $F(n,k)$.

5. Let $(u_n)$ be the sequence of integers defined by $u_1 = 39$, $u_2 = 45$ and $u_{n+2} = u_{n+1}^2 - u_n$. Prove that 1986 divides infinitely many terms of the sequence.



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