34-th Canadian Mathematical Olympiad 2002

March 27, 2002

1. Let S be a subset of $\{1,2,\dots,9\}$ such that all pairwise sums of elements in S are different. What is the maximum possible number of elements in S?

2. We call a positive integer n *practical* if every positive integer less than or equal to n can be written as the sum of distinct divisors of n. Prove that the product of two practical numbers is also practical.

3. Prove that for all positive real numbers a,b,c,
$$\frac{a^3}{bc}+\frac{b^3}{ca}+\frac{c^3}{ab} \geq a+b+c,$$
and determine when equality occurs.

4. Let Γ be a circle with radius r, and let A and B be distinct points on Γ such that $AB < \sqrt{3}r$. Let P be the point inside Γ such that triangle ABP is equilateral, and let the circle with center B and radius AB meet Γ again at C. The line CP meets Γ again at Q. Prove that $PQ = r$.

5. Determine all functions $f : \mathbb{N}_0 \to \mathbb{N}_0$ such that
$$xf(y)+yf(x) = (x+y)f(x^2+y^2) \qquad \text{for all } x,y \in \mathbb{N}_0.$$

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