Bulgarian Mathematical Olympiad 1964, IV Round

1. A $6n$-digit number is divisible by 7. Prove that if its last digit is moved at the beginning of the number (first position) then the new number is also divisible by 7. (5 points)

2. Find all possible n-tuples of reals: $x_1, x_2, \ldots, x_n$ satisfying the system:

$$\begin{cases} x_1 \cdot x_2 \cdots x_n = 1 \\ x_1 - x_2 \cdot x_3 \cdots x_n = 1 \\ x_1 \cdot x_2 - x_3 \cdot x_4 \cdots x_n = 1 \\ \ldots \\ x_1 \cdot x_2 \cdots x_{n-1} - x_n = 1 \end{cases}$$

(4 points)

3. There are given two intersecting lines g_1, g_2 and a point P in their plane such that $\angle(g_1, g_2) \neq 90^\circ$. Its symmetrical points on any random point M in the same plane with respect to the given planes are M_1 and M_2. Prove that:
   (a) the locus of the point M for which the point M_1, M_2 and P lies on a common line is a circle k passing intersecting point of g_1 and g_2.
   (b) the point P an orthocenter of the triangle, inscribed in the circle k sides of which lies at the lines g_1 and g_2.

   (6 points)

4. Let a_1, b_1, c_1 are three lines each two of them are mutually crossed and aren't parallel to some plane. The lines a_2, b_2, c_2 intersects the lines a_1, b_1, c_1 at the points a_2 in A, C_2, B_1; b_2 in C_1, B, A_2; c_2 in B_2, A_1, C respectively in such a way that A is the middle line of B_1C_2, B is the middle of C_1A_2 and C is the middle of A_1B_2. Prove that:
   (a) A is the middle of the B_2C_1, B is the middle of C_2A_1 and C is the middle of A_2B_1;
   (b) triangles $A_1B_1C_1$ and $A_2B_2C_2$ are the same. ($A_1B_1C_1A_2B_2C_2$ - is a prism).

   (5 points)

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