35-th Bulgarian Mathematical Olympiad 1986
Third Round

First Day

1. Determine all natural numbers n for which $3^n + 88$ is a perfect square.

2. Find all real values p for which the function

$$f(x) = x^2 + 2p|x - p| + |2p - 1|x - p^2$$

   takes only nonnegative values.

3. Let I be a point inside a cyclic quadrilateral $ABCD$. Let D_A, D_B, D_C denote the points symmetric to D with respect to the lines IA, IB, IC, respectively. Prove that if the lines AD_A, BD_B and CD_C are parallel, then I is the incenter of the triangle ABC.

Second Day

4. Prove that the equation

$$(\sqrt{x+1}+1)(\sqrt{x+16}+4) - (\sqrt{x+4}+2)(\sqrt{x+9}+3) = 0$$

   has no real roots.

5. In a given tetrahedron $A_1A_2A_3A_4$, I_k denotes the incenter of the face opposite to A_k ($k = 1, 2, 3, 4$). Show that if the lines A_1I_1, A_2I_2, A_3I_3 have a common point, then this point belongs to A_4I_4 as well.

6. From the arithmetic progression $1, 4, 7, \ldots, 100$, 19 numbers are selected. Prove that the sum of some two of the selected numbers divides 104.