30-th Bulgarian Mathematical Olympiad 1981

Third Round

First Day

1. On a chess tournament 12 players took part, and any two of them played exactly one match. It turned out after the tournament that the first three together gained three times more points then the last five. How did the match between the seventh and eighth player end?

2. Find all real numbers q such that for every positive p the equation

$$\log_2\left(1-\frac{3}{8}x-\frac{1}{2}x^2\right)=q\log_{1-\frac{3}{8}x-\frac{1}{2}x^2}2+p$$

has a root in the interval $(0,1)$.

3. Let m and n be two given segments ($m<n$). Construct a triangle ABC which satisfies the following conditions:

   (i) If the angle bisector at B intersects AC in E and the angle bisector at A in M, then the circumcircle of the triangle AEM lies on the line AB;

   (ii) $AE=m$ and $EC=n$.

Second Day

4. Show that if n is a positive integer for which $1+2^n+4^n$ is prime, then n is a power of 3.

5. Prove that if the lengths of the interior and exterior angle bisector at C in a triangle ABC are equal, then $AC^2+BC^2=4R^2$, where R is the circumradius of $\triangle ABC$.

6. The sphere inscribed in a given tetrahedron touches three of its faces at their orthocenters. Prove that the tetrahedron is regular.

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