Bulgarian Mathematical Olympiad 1965, III Round

First Day

1. On a circumference are written 1965 digits, It is known if we read the digits on the same direction as the clock hand is moving, resulting 1965-digit number will be divisible to 27. Prove that if we start reading of the digits from some other position the resulting 1965-digit number will be also divisible to 27. (7 points)

2. Find all real roots of the equation:

$$\sqrt{x^2-2p}+\sqrt{4x^2-p-2}=x$$

where p is real parameter. (points)

3. Prove that if α, β, γ are angles of some triangle then

$$A=\cos\alpha+\cos\beta+\cos\gamma<2$$

(6 points)

Second day

4. It is given an acute-angled triangle ABC. Perpendiculars to AC and BC drawn from the points A and B intersects in the point P. Q is the projection of P on AB. Prove that the arms of $\angle ACB$ cut from a line passing through Q and different from AB segment bigger than the segment AB. (7 Points)

5. Construct a triangle ABC by given side $AB=c$ and distances p and q from vertices A and B to the angle bisector of angle C. Express the area of the triangle ABC by c, p and q. (7 points)

6. Let P is not an external point to the tetrahedron $DABC$ different from the point D. Prove that from the segments PA, PB, PC can be chosen a segment that is shorter from some of the segments DA, DB, DC.

(6 points)

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