Bulgarian Mathematical Olympiad 1964, III Round

First Day

1. Find four-digit number: $\overline{xyzt}$ which is an exact cube of natural number if its four digits are different and satisfy the equations: $2x = y - z$ and $y = t^2$. (7 points)

2. Find all possible real values of k for which roots of the equation
$$(k+1)x^2 - 3kx + 4k = 0$$
are real and each of them is greater than -1. (7 points)

3. Find all real solutions of the equation:
$$x^2 + 2x\cos(xy) + 1 = 0$$
(7 points)

Second day

4. A circle k and a line t are tangent at the point T. Let M is a random point from t and MA is the second tangent to k. There are drawn a diameter AB and a perpendicular TC to AB (C lies on AB):
   (a) prove that the intersecting point P of the lines MB and TC is a midpoint of the segment TC;
   (b) find the locus of P when M is moving over the line t.

   (7 points)

5. In the tetrahedron $ABCD$ all pair of opposite edges are equal. Prove that the lines passing through their midpoints are mutually perpendicular and are axis of symmetry of the given tetrahedron. (7 points)

6. Construct a right-angled triangle by given hypotenuse c and an obtuse angle φ between two medians to the cathets. Find the allowed range in which the angle φ belongs (min and max possible value of φ).

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