Bulgarian Mathematical Olympiad 1963, III Round

First Day

1. From the three different digits x, y, z are constructed all possible three-digit numbers. The sum of these numbers is 3 times bigger than the number which all three digits are equal to x. Find the numbers: x, y, z.

    (7 points)

2. Solve the inequality:

$$\frac{1}{2(x-1)} - \frac{4}{x} + \frac{15}{2(x+1)} \geq 1$$

    (7 points)

3. If α, β, γ are the angles of some triangle prove the equality:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta + \cos\gamma = 1$$

    (6 points)

Second day

4. Construct a triangle, similar to a given triangle one if one of its vertices is same as a point given in advance and the other two vertices lie at a given in advance circle. (HINT: You may use circumscribed around required triangle circle)   (8 points)

5. A regular tetrahedron is cut from a plane parallel to some of its base edges and to some of the other non-base edges, non intersecting the given base line. Prove that:

    (a) the intersection is a rectangle;

    (b) perimeter ot the intersection doesn't depent of the situation of the cutting plane.

    (5 points)

6. Find dihedral line φ, between base wall and non-base wall of regular pyramid which base is quadrilateral if it is known that the radii of the circumscribed sphere bigger than the radii of the inscribed sphere.

    (7 points)