Bulgarian Mathematical Olympiad 1961, III Round

First Day

1. Let a and b are two numbers with greater common divisor equal to 1. Prove that that from all prime numbers which square don't divide the number: $a+b$ only the square of 3 can divide simultaneously the numbers $(a+b)^2$ and a^3+b^3. (7 points)

2. What relation should be between p and q so that the equation
$$x^4+px^2+q=0$$
have four real solutions forming an arithmetic progression? (6 points)

3. Express as a multiple the following expression:
$$A=\sqrt{1+\sin x}-\sqrt{1-\sin x}$$
if $-\frac{7\pi}{2}\leq x\leq -\frac{5\pi}{2}$ and the square roots are arithmetic. (7 points)

Second day

4. In a circle k are drawn the diameter CD and from the same half line of CD are chosen two points A and B. Construct a point S on the circle from the other half plane of CD such that the segment on CD, defined from the intersecting point M and N on lines SA and SB with CD to have a length a. (7 points)

5. In a given sphere with radii R are situated (inscribed) six same spheres in such a way that each sphere is tangent to the given sphere and to four of the inscribed spheres. Find the radii of inscribed spheres. (7 points)

6. Through the point H, not lying in the base of a given regular pyramid is drawn a perpendicular to the plane of the base. Prove that the sum from the segments from H to intersecting points of the perpendicular given to the planes of all non-base sides of the pyramid doesn't depend on the position of H on the base plane. (6 points)

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