9-th Bosnia and Hercegovina Mathematical Olympiad 2004

Sarajevo, May 8–9

First Day

1. Two circles are internally tangent to a circle with center O at points S and T and intersect each other at points M and N, with N closer to ST. Prove that the lines OM and ON are perpendicular if and only if the points S,N and T are collinear.

2. Determine whether there exists a triangle whose sides have integral lengths and whose area is 2004.

3. Positive numbers a,b,c satisfy $abc = 1$. Prove the inequality

$$\frac{ab}{a^5+b^5+ab}+\frac{bc}{b^5+c^5+bc}+\frac{ca}{c^5+a^5+ca}\leq 1.$$

Second Day

4. On a tournament with 16 participating teams 55 matches were played. Prove that there exist three teams, no two of which played a match.

5. For $0 \leq x < \frac{\pi}{2}$ and arbitrary real numbers a,b prove the inequality

$$a^2 \tan x \cos^{\frac{1}{3}} x + b^2 \sin x \geq 2xab.$$

6. In the plane are given a triangle ABC and a parallelogram $ASCR$. The line through B parallel to CS meets line AS at M and line CR at P, while the line through B parallel to AS meets AR at N and CS at Q. Prove that the lines RS,MN and PQ are concurrent.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
Provided by Salem Malikić
www.imomath.com