Flanders Mathematical Olympiad 1999

Final Round

1. Find all six-digit natural numbers $\overline{abcdef}$ with $a \neq 0$ and $d \neq 0$ such that

$$\overline{abcdef} = \overline{def}^2.$$

2. Let MN be a diameter of a circle with a given radius and AB a chord of a given length on this circle. Suppose that AB neither coincides nor is perpendicular to MN. Let P be the midpoint of AB and let C and D be the orthogonal projections of A and B on MN. Prove that $\angle CPD$ is independent of the choice of AB.

3. Determine all functions $f, g : \mathbb{R} \to \mathbb{R}$ satisfying the following conditions:

   (i) $2f(s) - g(s) = f(t) - t$ for all real s and t;

   (ii) $f(s)g(s) \geq s + 1$ for all real s.

4. Consider integers a, b, m, n greater than 1 such that $a^n - 1$ and $b^m + 1$ are both prime numbers. Give as much information as possible on a, b, m, n.