Belgium Flanders Mathematical Olympiad 2004

1 Consider a triangle with side lengths 501m, 668m, 835m. How many lines can be drawn which divide triangle into two parts of equal area and perimeter?

2 Two bags contain some numbers, and the total number of numbers is prime. When we tranfer the number 170 from bag 1 to bag 2, the average in both bags increases by one. If the total sum of all numbers is 2004, find the number of numbers.

3 A car has a 4-digit integer price, which is written digitally (i.e. in digital numbers, like on digital watch). While the salesmen was absent, the buyer turned the price upside down and got the car for 1626 less. What was initial price of the car?

4 Each cell of a beehive is constructed from a right regular 6-angled prism, openned at the bottom and closed on the top by a regular 3-sided pyramidical mantle. The edges of this pyramid are connected to three of the rising edges of the prism and its apex T is on the perpendicular line through the center O of the base of the prism (see figure). Let s denote the side of the base, h the height of the cell and θ the angle between the line TO and TV.
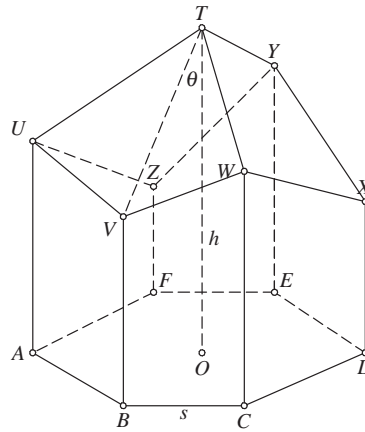
(a) Prove that the surface of the cell consists of 6 congruent trapezoids and 3 congruent rhombi.

(b) Prove that the total surface area of the cell is given by the formula

$$6sh - \frac{9s^2}{2\tan\theta} + \frac{s^2 3\sqrt{3}}{2\sin\theta}.$$