1-st Balkan Mathematical Olympiad

Athens, Greece – May 6-10, 1984

1. If $a_1, a_2, \dots, a_n$ $(n \geq 2)$ are positive real numbers with $a_1 + a_2 + \cdots + a_n = 1$, prove that
$$\frac{a_1}{1 + a_2 + a_3 + \cdots + a_n} + \cdots + \frac{a_n}{1 + a_1 + a_2 + \cdots + a_{n-1}} \geq \frac{n}{2n-1}.$$
(Greece)

2. Let $ABCD$ be a cyclic quadrilateral and H_A, H_B, H_C, H_D be the orthocenters of the triangles BCD, CDA, DAB, ABC, respectively. Prove that the quadrilaterals $ABCD$ and $H_AH_BH_CH_D$ are congruent. *(Romania)*

3. Prove that for every positive integer m there exists $n > m$ such that the decimal representation of 5^n can be obtained from the decimal representation of 5^m by adding several digits to the left. *(Bulgaria)*

4. Given positive real numbers a, b, c, find all real solutions to the system
$$\begin{aligned} ax + by &= (x - y)^2, \\ by + cz &= (y - z)^2, \\ cz + ax &= (z - x)^2. \end{aligned}$$
(Romania)

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