# 10-th Asian–Pacific Mathematical Olympiad 1998

1. Let $\mathscr{F}$ be the set of all $n$-tuples $(A_1, A_2, \ldots, A_n)$, where each $A_i$ is a subset of $\{1, 2, \ldots, 1998\}$. Evaluate

$$\sum_{(A_1,A_2,\ldots,A_n)\in\mathscr{F}} |A_1 \cup A_2 \cup \cdots \cup A_n|.$$

2. Show that for any positive integers $a$ and $b$, $(36a+b)(a+36b)$ cannot be a power of 2.

3. If $a, b, c$ are positive real numbers, prove the inequality

$$\left(1+\frac{a}{b}\right)\left(1+\frac{b}{c}\right)\left(1+\frac{c}{a}\right) \geq 2\left(1+\frac{a+b+c}{\sqrt[3]{abc}}\right).$$

4. In a triangle $ABC$, $D$ is the foot of the altitude from $A$. Let $E$ and $F$ be points on a line through $D$, different from $D$, such that $AE \perp BE$ and $AF \perp CF$, and let $M$ and $N$ be the midpoints of the segments $BC$ and $EF$, respectively. Prove that $AN$ is perpendicular to $NM$.

5. Determine the largest integer $n$ which is divisible by all positive integers that are less than $\sqrt[3]{n}$.



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