12-th Asian–Pacific Mathematical Olympiad 2000

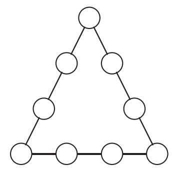
1. Compute the sum $S = \sum_{i=0}^{101} \frac{x_i^3}{1 - 3x_i + 3x_i^2}$ for $x_i = \frac{i}{101}$.

2. The numbers $1, 2, \dots, 9$ are written in the nine circles on the picture (one number in each circle) so that:

   (i) The sums of the four numbers on each side of the triangle are equal;

   (ii) The sums of the squares of the four numbers on each side are equal.

   Find all ways in which this can be done.



3. In a triangle ABC, the median and the angle bisector at A meet the side BC at M and N respectively. The perpendicular at N to NA meets MA in Q and BA in P, and the perpendicular at P to BA meets AN produced in O. Prove that QO is perpendicular to BC.

4. Let n, k be positive integers with $n > k$. Prove that

$$\frac{1}{n+1} \cdot \frac{n^n}{k^k (n-k)^{n-k}} < \frac{n!}{k!(n-k)!} < \frac{n^n}{k^k (n-k)^{n-k}}.$$

5. Given a permutation $(a_0, a_1, \dots, a_n)$ of the sequence $0, 1, \dots, n$, a transposition of a_i $(i > 0)$ with a_j is called *legal* if $a_i = 0$ and $a_{i-1} + 1 = a_j$. The permutation $(a_0, a_1, \dots, a_n)$ is called *regular* if after a number of legal transpositions it becomes $(1, 2, \dots, n, 0)$. For which numbers n is the permutation $(1, n, n-1, \dots, 3, 2, 0)$ regular?

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